Glacier Model

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1 About this document

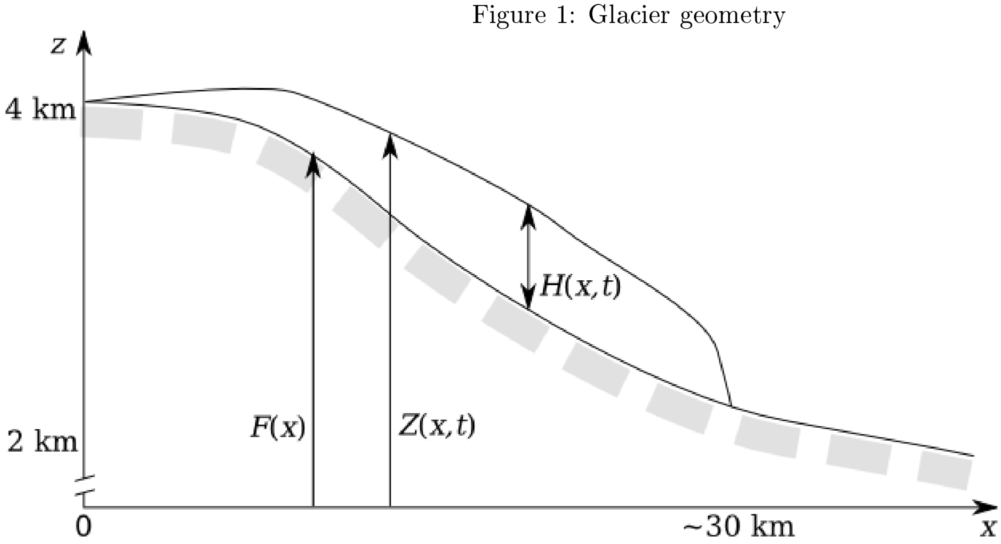
This document describes the physics behind the glacier dynamics in the PhET sim. It is currently under construction. Sections 2 and 3 are more or less complete. Later sections are still in the works.

2 Glacier geometry

The variables describing the shape and size of the glacier are shown in figure 1. The spatial dimensions are labeled x (horizontal) and z (vertical), and have units of meters. $F(x)$ defines the elevation of the floor of the valley, and $x = 0$ is set at the point of maximum elevation where the glacier begins.

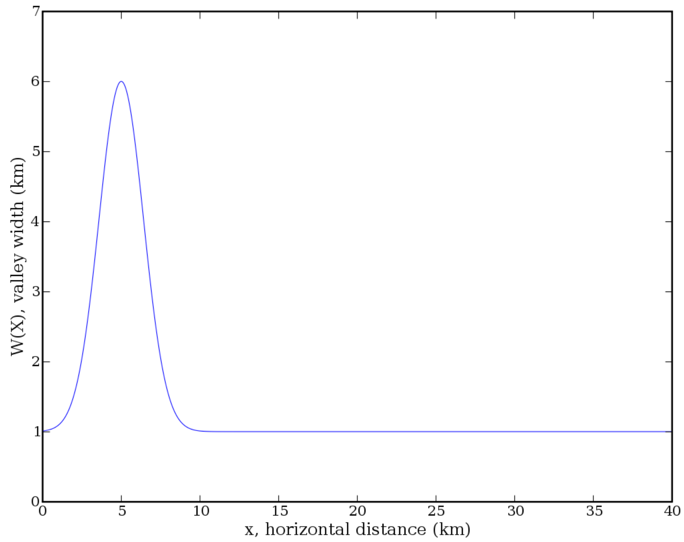
The thickness of the glacier is $H(x,t)$, where t is time in years. The elevation of the glacier's surface is $Z(x,t) = F(x) + H(x,t)$. The width of the valley is $W(x)$, and is pre-defined to have some profile similar to

Figure 1: Glacier geometry



the one shown in figure 2. The valley width is largest at the highest elevations (reflecting the large collection

Figure 2: Valley width (example)



area at the glacial headwaters) and is constant along the long valley down which the glacier will extend. We make the approximation of vertical valley walls, so that the cross-sectional area of the glacier is HW at all times.

The time dependence of the glacier's thickness, $H(x,t)$, is governed by four processes:

1. ice flow due to viscous deformation,
2. ice flow due to sliding,
3. ice accumulation due to snowfall, and
4. ice ablation (mass loss due to melting and sublimation).

These processes are discussed in sections 4 and 5.

3 Glacier object interface

The glacier is completely described by its thickness, $H(x,t)$, and internal ice velocity field, $\mathbf{V}(x,z,t)$. The thickness is a scalar and velocity is a two-component vector ($\mathbf{V} = V_x\hat{x} + V_z\hat{z}$, where $\hat{x}$ and $\hat{z}$ are unit vectors).

Given some fixed valley geometry ($F(x)$ and $W(x)$), H and $\mathbf{V}$ are completely determined by the present and past climate conditions. For our purposes, the climate is completely described by two independent parameters which may vary in time: temperature $T_0(t)$ and precipitation $P_0(t)$. The subscript 0 indicates that these quantities describe temperature and precipitation at some reference elevation (their z-dependence will follow from these values).

The user may specify the current and future values of both T_0 and P_0. If future values are not specified, the climate will remain constant in time. If future values are specified (along with some target time for the future values to apply), T_0 and P_0 will change linearly from current to future values. The only other climate change a user can make is to reset the glacier to its equilibrium thickness, which is a direct function of the present values of T_0 and P_0.

The values for $H(x,t)$ and $\mathbf{V}(x,z,t)$ depend on the integrated climate history; that is, they depend on the parameters $T_0(t)$ and $P_0(t)$ for all times previous to the present. This dependence is developed in the following sections.

4 Ice flow

Glacial ice moves down-valley by two processes, sliding and viscous deformation, whose velocities (u_s and u_d, respectively) add to form the total velocity of ice flow.

Viscous deformation occurs due to stress from the weight of overlying ice. This stress is given by

$$\tau_g = \rho g \frac{dH}{dx}(H - \xi) \tag{1}$$

where ρ is the density of ice (about 10^3 kg/m^3), g is gravitational acceleration (9.8 m/s^2), and ξ is the height above the valley floor ($\xi = z - F$). Glenn's Flow Law specifies the vertical profile of the deformation velocity as follows:

$$\frac{du_d}{d\xi} = A\tau^3 \tag{2}$$

where A is a temperature-dependent constant (about 6.8×10^{-24} Pa$^{-3} \cdot$ s^{-1} at temperatures appropriate for temperate alpine glaciers). Using equation 1 in equation 2, and integrating from the base to the top of the glacier ($\xi = 0$ to $\xi = H$) yields the deformation velocity as a function of vertical distance in the ice:

$$u_d(\xi) = A\left(\rho g \frac{dH}{dx}\right)^3 \left(H^3\xi - \frac{3}{2}H^2\xi^2 + H\xi^3 - \frac{1}{4}\xi^4\right). \tag{3}$$

The average deformation velocity is then

$$\bar{u}_d = \frac{1}{5}A\left(\rho g \frac{dH}{dx}\right)^3 H^4. \tag{4}$$

Finally, it is convenient to write the deformation velocity in terms of its vertical-average:

$$u_d(\xi) = \bar{u}_d \left(\zeta - \frac{3}{2}\zeta^2 + \zeta^3 - \frac{1}{4}\zeta^4 \right) \tag{5}$$

where the dimensionless $\zeta = \xi / H$.

The physics of glacier sliding is not well understood. Sliding velocity u_s can, however, be fairly accurately modeled (Kessler et al., 2006) with

$$u_s = U_c e^{(1-\tau_c/\tau_g)} \tag{6}$$

where U_c is a characteristic sliding value (taken to be 20 m/yr) and τ_c is a characteristic gravitational stress (taken to be 1 bar, or 10^5 Pa); τ_g is the actual gravitational stress from equation 1.

Given the velocities calculated above, the net flux of ice volume through a vertical plane (called the discharge, Q) is

$$Q(x,t) = W(x)H(x,t)\left[\bar{u}_d(x,t) + u_s(x,t)\right] \tag{7}$$

where $\bar{u}_d$ and u_s are calculated from equations 4 and 6, respectively.

5 Mass balance

The thickness of the glacier is also affected by ice accumulation and ablation (melting and sublimation). These processes are governed by the "mass balance," a function which describes the time rate of change of the glacier's height depending on the elevation. The mass balance, $b(z)$, is often defined to be a linear function of elevation with a maximum cutoff value of $b_{\max}$:

$$b(z) = \min\left\{\gamma\left(z - z_{\text{ela}}\right), \quad b_{\max}\right\} \tag{8}$$

where γ, z_{ela} and $b_{\max}$ are set to reflect the precipitation and melting conditions of the local environment. Typical values may be $\gamma \approx 10$, $z_{\text{ela}} \approx 2800$m, and $b_{\max} \approx 2$m/yr. These quantities may be seen in figure 3.

References

Kessler, M., Anderson, R. S., & Stock, G., 2006. Modeling topographic and climatic control of east-west asymmetry in Sierra Nevada glacier length during the Last Glacial Maximum, *J. Geophys. Res.*, **111**(F2).

Figure 3: Mass balance (example)